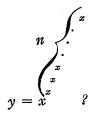
## Reprinted from Mathematics Magazine 40.5(1967)284.

## **Exponential Derivative**

**653.** [March, 1967] Proposed by Sam Newman, Atlantic City, New Jersey. What is dy/dx of



II. Solution by Stanley Rabinowitz, Far Rockaway, New York.

Let  $f_n$  denote the function  $x^{x cdots x}$  (where there are nx's).  $(df_n/dx) = f_n f_{n-1}/x + f_n (\ln x) df_{n-1}/dx$ . Using this formula, one easily finds that

$$\frac{df_n}{dx} = f_n f_{n-1}/x + f_n f_{n-1} f_{n-2} (\ln x)/x + f_n f_{n-1} (\ln x)^2 \frac{df_{n-2}}{dx}.$$

Continuing to substitute, one gets by induction

$$\frac{df_n}{dx} = \sum_{j=1}^k \left[ \frac{(\ln x)^{j-1}}{x} \prod_{i=0}^j f_{n-i} \right] + \left[ \prod_{i=0}^{k-1} f_{n-i} \right] (\ln x)^k \frac{df_{n-k}}{dx}.$$

When k=n-1, we have

$$\frac{df_n}{dx} = \sum_{j=1}^{n-1} \left[ \frac{(\ln x)^{j-1}}{x} \prod_{i=0}^{j} f_{n-i} \right] + (\ln x)^{n-1} \prod_{i=0}^{n-2} f_{n-i}$$